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# Modulus number and reference tangent modulus of clays in the Red River delta

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ABSTRACT: This study presents an investigation of Janbu modulus number ($m$) and reference tangent modulus ($E_{oed}^{ref}$) of clays in the Red River delta. These are important input parameters in different nonlinear constitutive soil models. Standard consolidation test results of 190 specimens obtained from 12 test sites over the delta were analyzed. The $m$ was found to range mostly from 8 through 30 and agrees well with typical values of soft to stiff clays in the literature. The $m$ value determined from strain ($\varepsilon_v$) versus applied stress ($\sigma'_v$) was found almost equal to that from the equation $m = 2.3(1+e_0)/C_c$ but was on average 1.25 times larger than the value determined from the tangent modulus ($M_t$) versus applied stress ($\sigma'_v$). This can mainly be attributed to natural structure and heterogeneity of intact soils. The $E_{oed}^{ref}$ value of the clays was found to range from 1.0 to 6.0 MPa and is similar to the value range of Bangkok clays. The $E_{oed}^{ref}$ value was found to have no clear correlations with $m$, cone resistance ($q_t$) but a relatively good nonlinear correlation with $C_c$.

## 1. INTRODUCTION

The Red River Delta (RRD) is the second largest delta in Vietnam and it plays a significant role in the economic progress of the country with regard to both agricultural and industrial sectors (GSO 2021). The recent rapid development has required expanding the infrastructure in the delta area (e.g., highways, harbours, industrial plants, and logistic facilities).

Understanding and using deformation properties of soil layers in the delta correctly, especial of clay layers, are very important for the assessing settlement of the infrastructure engineering projects. In fact, very few studies reported geotechnical properties of clayey soils at some places in the delta (e.g., Hien and Giao, 2010, Phuc and Giao, 2019). Nguyen and Khin (2023) first attempted to characterize some compressibility characteristics of clayey soil in the whole delta using a database of laboratory and field tests at twelve test sites. They indicated that the delta comprises soft to medium stiff clay layers of Holocene period (9,000 years BP).

The consolidation settlement of clayey soil is often estimated by using the Terzaghi's traditional consolidation theory with the use of compression index ($C_c$), *in-situ* void ratio ($e_0$). The settlement can also be estimated by using the Janbu's tangent modulus method (Holtz 1991). This paper presents a study on the Janbu modulus number ($m$) and the reference tangent modulus ($E_{oed}^{ref}$) (for hardening soil model) of clay samples obtained from shallow through about 30 m depths from 12 test sites across the delta.

## 2. DEFORMATION PARAMETERS

### *2.1 Modulus number (m)*

From the applied stress ($\sigma'_v$) versus cumulative vertical strain ($\varepsilon_v$) data of the consolidation test, Janbu (1963, 1998) recommended a tangent modulus ($M_t$) expressed by Eq. 1.

$$M_t = \frac{d\sigma'_v}{d\varepsilon_v} = m\sigma_a\left(\frac{\sigma'_v}{\sigma_a}\right)^{1-j} \tag{1}$$

where $m$ = modulus number; $j$ = stress exponent; $\sigma'_v$ = effective stress, $\sigma_a$ = reference stress, 100 kPa. The stress exponent ($j$) depends on soil type and ranges from 0 through 1.

For cohesive soil, $j$ = 0, therefore $M_t = m\sigma'_v$, $m$ is the slope of the relative linear portion of the $M_t$-$\sigma'_v$ curve when $\sigma'_v$ is larger than the preconsolidation tress ($\sigma'_p$) (Fig. 1a).

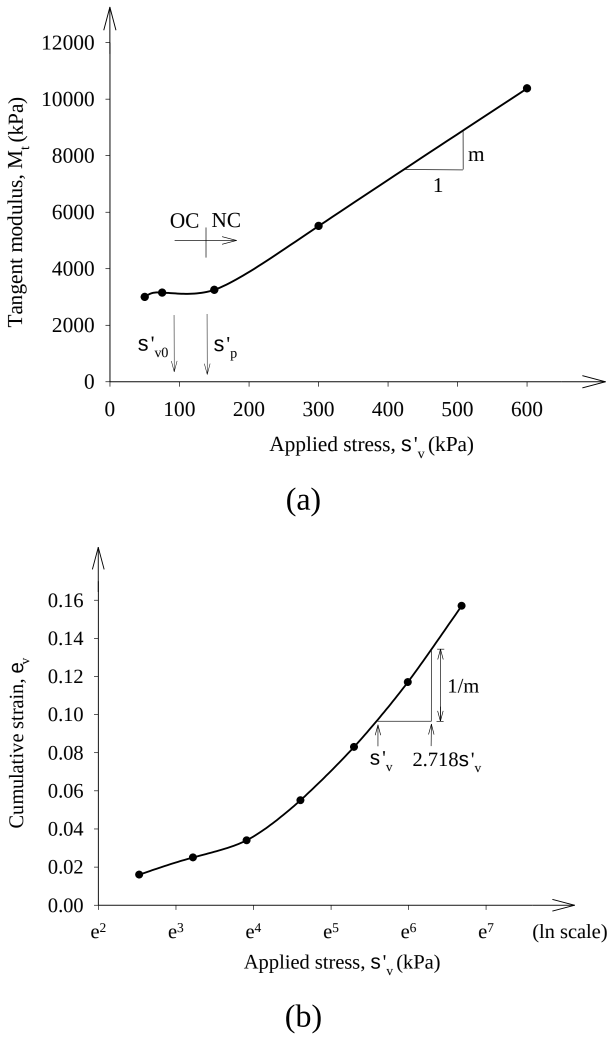



Figure 1. Determination of m value: (a) from applied stress-tangent modulus curve (Janbu, 1998); (b) from applied stress versus cumulative strain curve (Fellenius, 2023)

From Eq. (1), the vertical strain ($\varepsilon_v$) of a soil element can be derived as follows (Janbu 1998, Holtz 1991, Fellenius 2023).

$$\varepsilon_v = \frac{1}{m_r}\ln\left(\frac{\sigma'_p}{\sigma'_{v0}}\right) + \frac{1}{m}\ln\left(\frac{\sigma'_v}{\sigma'_p}\right) \tag{2}$$

where $m_r$ is the modulus number in the recompression range and $m_r$ is the slope of the curve in the OC range ($\sigma'_v < \sigma'_p$).

Based on Eq. (2), $m$ can also be determined from the slope (1/$m$) of the applied stress ($\sigma'_v$ in natural logarithm scale) vesus cumulative vertical strain ($\varepsilon_v$) curve in the NC range (Fig. 1b).

Equalizing the $\varepsilon_v$ from the Terzaghi's conventional consolidation theory and the $\varepsilon_v$ by Eq. (2) for the NC range, the following equation is obtained (Holtz 1991, Fellenius 2023).

$$m = \ln 10 \frac{1+e_0}{C_c} \tag{3}$$

where $e_0$ = the in-situ void ratio and $C_c$ = compression index of the soil.

### *2.2 Reference tangent modulus ($E_{oed}^{ref}$)*

The hardening soil model (Schanz et. al. 1999) is used in many commercial software, e.g., Plaxis and FLAC. In this soil model, one of the key calculated parameters is the oedometer one-dimensional tangent modulus ($E_{oed}$).

$$E_{oed} = E_{oed}^{ref}\left(\frac{\sigma'_v + c\cot\phi}{p^{ref} + c\cot\phi}\right)^n \tag{4}$$

where $E_{oed}^{ref}$= reference (oedometer) tangent modulus at $\sigma'_v = p^{ref}$ = 100 kPa

$n$ = power for stress-level dependency of compressibility

$c$ = cohesion intercept

$\phi$ = friction angle.

Eq. (4) is essentially an extension of Eq. (1) with $n$ = (1- $j$) and the inclusion of $c\cot\phi$.

$E_{oed}^{ref}$ is an important input parameter of the soil model. The value can be determined graphically from the stress-strain curve of a consolidation test as illustrated in Figure 2.

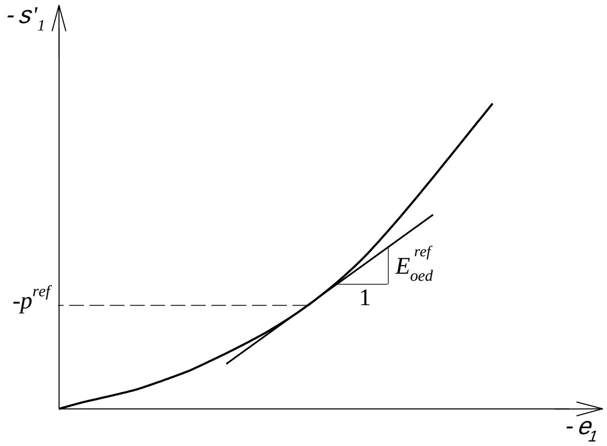



Figure 2. Determination of $E_{oed}^{ref}$ from the stress-strain curve (after Plaxis 2020)

## 3. STUDY SITES AND SOIL PROFILES

### 3.1 Study sites

In this study, geotechnical data from twelve test sites in the RRD in the North Vietnam were analyzed. All the sites were construction sites of civil and industrial projects, where extensive field and laboratory tests had been carried out during the site investigation stage. Figure 3 shows the abbreviated name and location of each site. Four sites (DVIZ, VSIP, KC, and ND TPP) were also the research sites of the first author investigating the consolidation characteristics of the clays in the delta. Details of the project names, site coordinates, and field and laboratory tests were described in detail in Nguyen and Khin (2023).

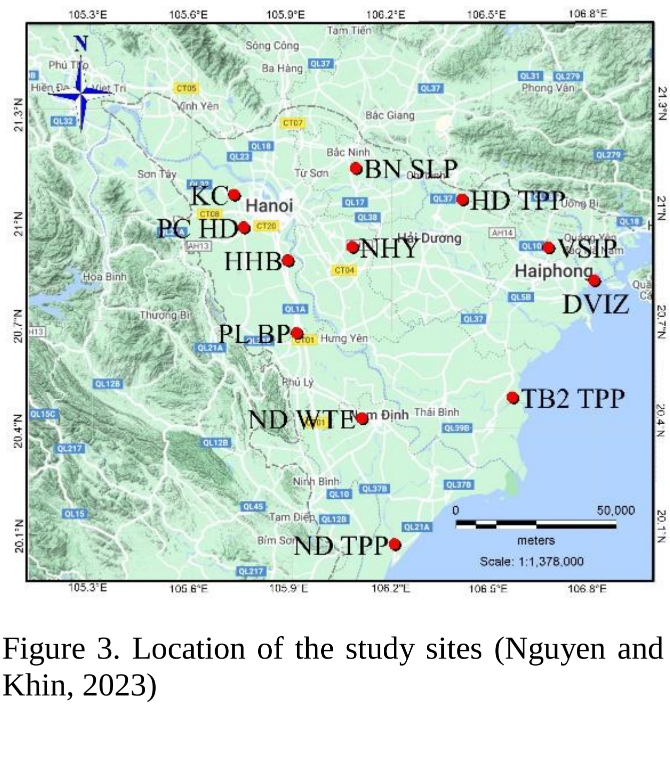



Figure 3. Location of the study sites (Nguyen and Khin, 2023)

### 3.2 Soil profiles

Table 1 provides depth, number of test samples, soil description of clay layers from the twelve sites and Figure 4 shows CPT $q_t$ -profiles in clayey layers from 10 sites in the database. The $q_t$-values in sandy interbedded layers were removed. The vertical dashed lines delineate the boundaries of compressibility ranges of clayey soil (Mayne and Kulhawy 1990).

(i) Very soft: $q_t < 0.25$ MPa
(ii) Soft: $0.25 \text{ MPa} \leq q_t < 0.5$ MPa
(iii) Medium stiff: $0.5 \text{ MPa} \leq q_t < 1.0$ MPa
(iv) Stiff: $1.0 \text{ MPa} \leq q_t < 2.0$ MPa
(iv) Very stiff: $q_t \geq 2.0$ MPa

The relatively linear $q_t$ profiles indicate that the clay layers are rather homogeneous and were formed under relatively static depositional environments. The profiles of overconsolidation ratio (*OCR*) and compression index ($C_c$) indicated that the clays in the delta are typically soft to medium stiff in the upper depths and medium stiff to stiff in the lower depths at some places (Nguyen and Khin 2023). They were typically normally consolidated to slightly overconsolidated with *OCR* mostly ranging from 1.0 through 2.0.

## 4. DEFORMATION PARAMETERS

### 4.1 Modulus number (m)

Standard consolidation test (ASTM D2435–11) with procedure B was conducted on a total of 190 clay specimens from the 12 sites. Each test comprised seven loading steps of 12.5, 25, 50, 100, 200, 400, and 800 kPa. The in-situ effective overburden stress ($\sigma'_{v0}$) and the void ratio ($e_0$) of the specimens ranged from 25 through 350 kPa and from 0.5 through 1.8, respectively. The modulus number ($m$), the preconsolidation stress ($\sigma'_p$), and compression index ($C_c$) were determined from each test. For comparison purpose, the $m$ value was determined from the $M_t$ - $\sigma'_v$ curve (c.f., Figure 1a), from $\varepsilon_v$ - $\sigma'_v$ curve (c.f. Fig. 1b) and from Eq. (3). In total, 20 outliers of $m$ and $C_c$ (very small or very large) were removed from the dataset.

Test results from the data set indicated that the modulus number ($m$) increased with depth and ranged mostly from 5 through about 30 (with a few values of between 30 and 40), which agrees well with the typical range of m for soft to stiff clays (Janbu 1998, Holtz 1991).

Table 1. Main type of clay layers in the database

| No. | Site name | Depths (m) | No. of sample | Soil description |
|---|---|---|---|---|
| 1 | KC | 7.8-25 | 18 | L3: Clay to silty clay, medium stiff; L5: Clay to silty clay, stiff |
| 2 | HHB | 2.3-31.8 | 19 | L3: Clay to silty clay, soft to medium stiff |
| 3 | PL BP | 4.4-23.6 | 10 | L2: Clay to silty clay, soft to medium stiff; L4: Clay – organic soil, soft to medium stiff |
| 4 | HD TPP | 2-34.1 | 20 | L3: Clay to silty clay, medium stiff; L5: Clay to silty clay, stiff |
| 5 | BN SLP | 1.7-33.9 | 25 | L2: Clay to silty clay, soft to medium stiff; L4: Clay to silty clay, stiff to very stiff |
| 6 | VSIP | 7-17 | 28 | L2: Clay to silty clay, soft; L4: Clay to silty clay, medium stiff |
| 7 | DVIZ | 8-25 | 14 | L3: Clay to silty clay, Soft to medium; L4: Silty clay and clayey silt, stiff |
| 8 | TB2 TPP | 6.4-34.4 | 10 | L5: Clay to silty clay, soft to medium stiff; ALL: Clay to silty clay, stiff |
| 9 | ND TPP | 7.3-19.5 | 14 | L2: Clay to silty clay, soft to medium stiff. |
| 10 | ND WTE | 1.4-35.7 | 12 | L2: Clay to silty clay, Soft to medium stiff; L4: Clay to silty clay, medium stiff to stiff. |
| 11 | PC HD | 2.4-15.9 | 12 | 2-10.0: Clay to silty clay, medium to stiff; 10.0 – 16.0: Silty clay, stiff to very stiff |
| 12 | NHY | 2.2-26.2 | 8 | 1.0-6.0 m: Sandy lean clay, soft to medium; 14.0 – 16.0 m: Lean clay, medium stiff |

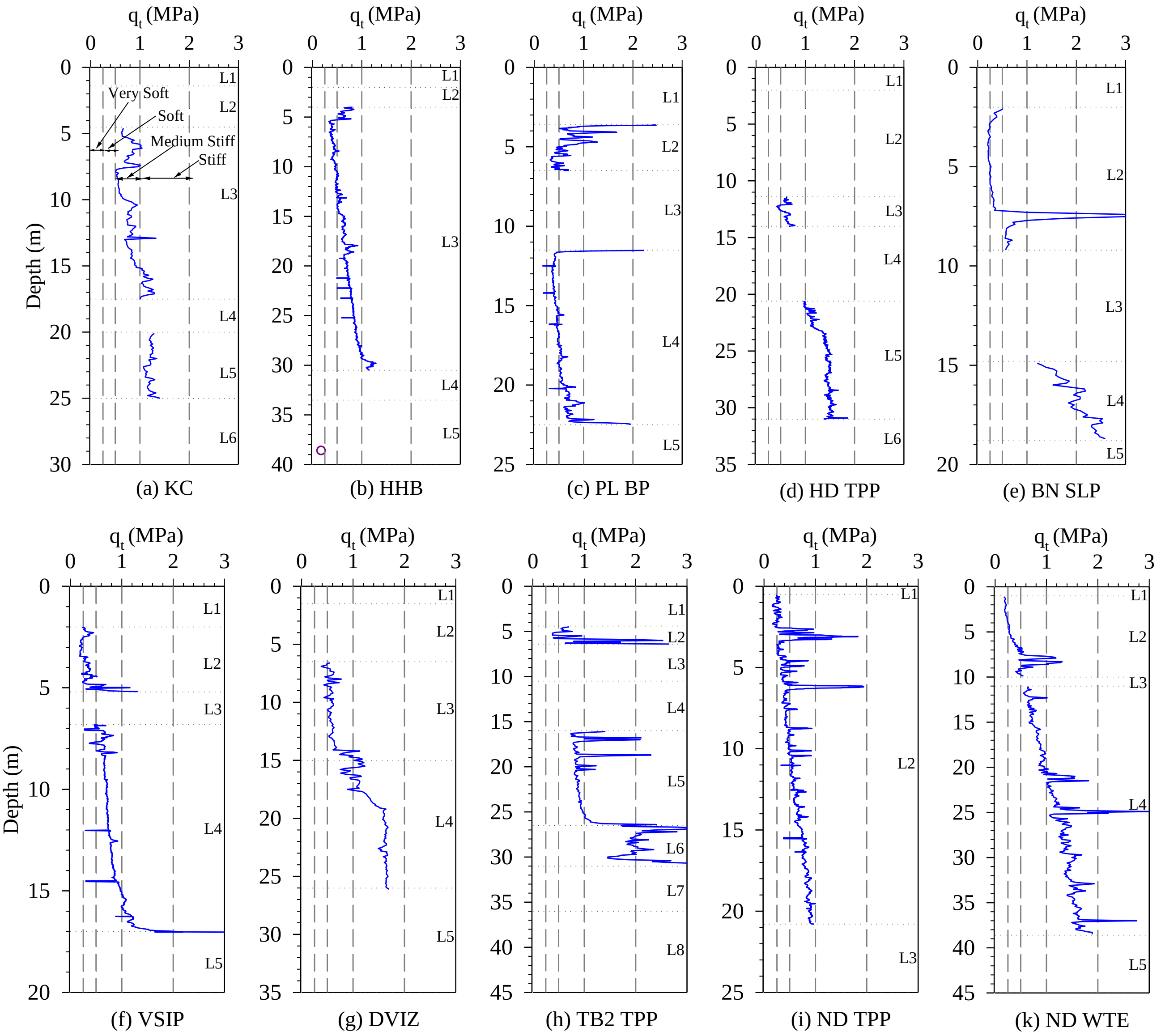
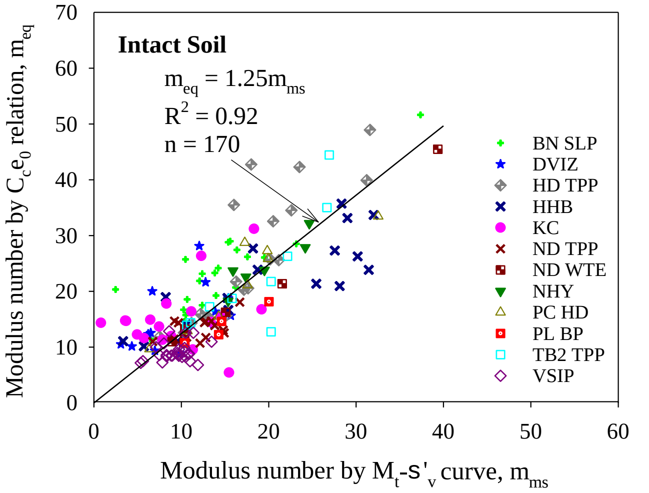
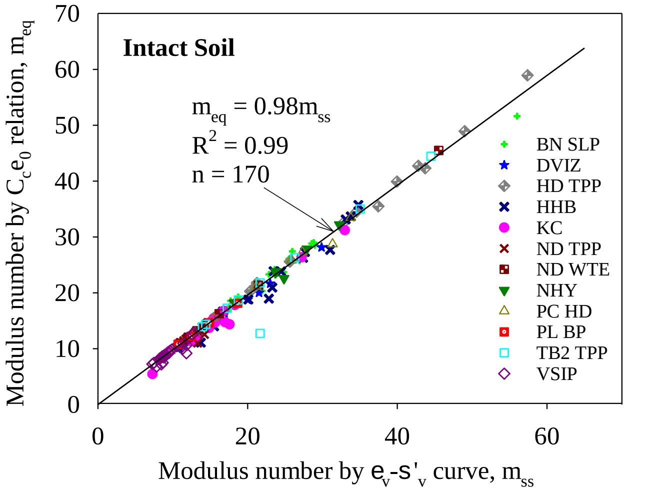



Figure 4. CPT $q_t$ profiles in clay layers at ten sites (Nguyen and Khin, 2023)

The $m$ values determined from $M_t$ - $\sigma'_v$ curve (herein named $m_{ms}$) and from $\varepsilon_v$ - $\sigma'_v$ curve ($m_{ss}$) are compared with the values back-calculated using the $C_c e_0$-relation (c.f., Eq. 3) ($m_{eq}$) as shown in Figures 5a and 5b, respectively. Interestingly, the values obtained from the $C_c e_0$-relation ($m_{eq}$) are almost equal to the values determined from $\varepsilon_v$ - $\sigma'_v$ curve ($m_{ss}$) and the correlation has a very high $R^2$ value (Fig. 5b). However, $m_{eq}$ values are on average 1.25 times larger than the values obtained from the $M_t$ - $\sigma'_v$ curve ($m_{ms}$) and the comparison shows a considerable scatter of values. Theoretically, the ratio of $m_{eq}/m_{ms}$ should be very closed to 1.0 as the ratio of $m_{eq}/m_{ss}$.




(a)




(b)

Figure 5. Comparison of $m$ obtained from: (a) $M_t$ - $\sigma'_v$ curve and Eq. (3); (b) $\varepsilon_v$ - $\sigma'_v$ curve and Eq. (3)

There are several possible reasons that resulted in scattered $m_{sm}$ – $m_{eq}$ correlation and the ratio of $m_{eq}/m_{sm}$ > 1. The first key reason is that $M_t = \Delta\sigma'_v/\Delta\varepsilon_v$ is very sensitive to small changes in $\varepsilon_v$. In fact, for the incremental loading method (ASTM D2435–11), $\Delta\sigma'_v$ and $\Delta\varepsilon_v$ are taken as stress and strain increments after each loading step. Thus any imprecisions in $\Delta\varepsilon_v$ measurement would result in large variation of $M_t$ and therefore $m$ value, causing the scattered correlation shown in Fig. 5a.

The second key reason may be attributed to the natural structure and homogeneity of the clays. Figure 6 shows three typical $M_t$ - $\sigma'_v$ curves at BN SLP site on which the $m$ value, $R^2$ value of the linear portion and the intercept of the linear line ($M_{t0}$) are presented. The figure shows that there exists a significant value of $M_{t0}$ for each curve, implying that the equation $M_t = m\sigma'_v$ for natural clays is rather ideal (i.e., the intercept is zero). The $M_{t0}$ values of the data set (Figure 7) show that most specimens resulted in $M_{t0} > 0$ while some specimens resulted in $M_{t0} < 0$.

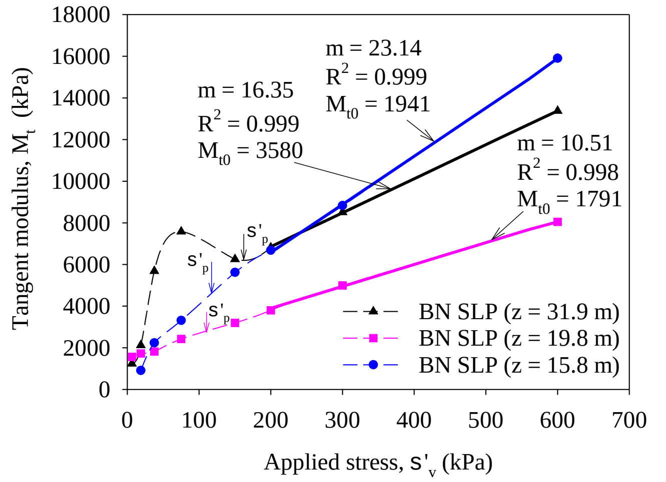



Figure 6. Variation of $M_t$ of some typical specimens

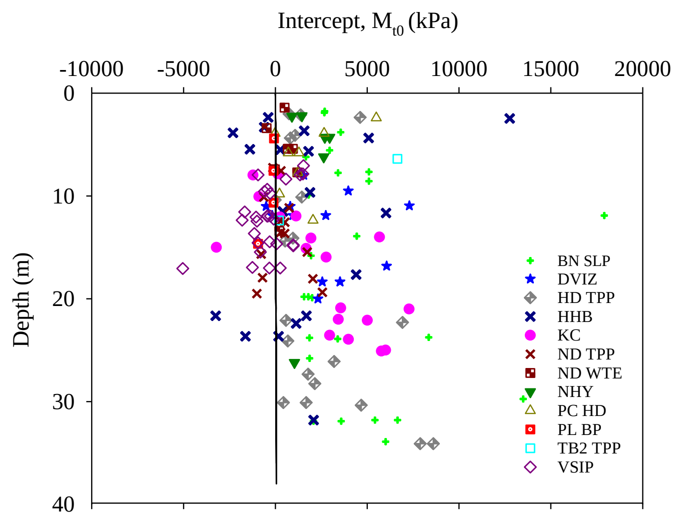



Figure 7. Variation of $M_{t0}$ with depth

To further investigate the influence of natural structure of clays, the consolidation test was carried out on remolded specimens from KC site and Lach Huyen Port, Berths 5 and 6 (LH 5 and 6), Hai Phong City. In total, there were y remolded specimens (7 at LH 5 and 6 and 4 at KC). Especially, at LH 5 and 6 site, another six intact specimens (at almost the same depths with the remolded ones) were also tested to make pairs for comparison purpose. For the remolded specimens, the soil was thoroughly remolded at the natural water content. The test results from the pairs indicates that the remolded specimens resulted in *m* values of 1.0 to 1.5 times larger the values from the intact specimens. The remolded curves are relatively linear in the whole range of applied pressure whereas the curves of intact specimens vary irregularly up to $\sigma'_p$ and the $M_{t0}$ of remolded specimens was typically smaller that from the intact specimens. For example, Figure 8 shows a comparison of two $M_t$ - $\sigma'_v$ curves of intact and remolded specimens from clay samples at 14.1 m and 14.3 m at LH 5 and 6 site.

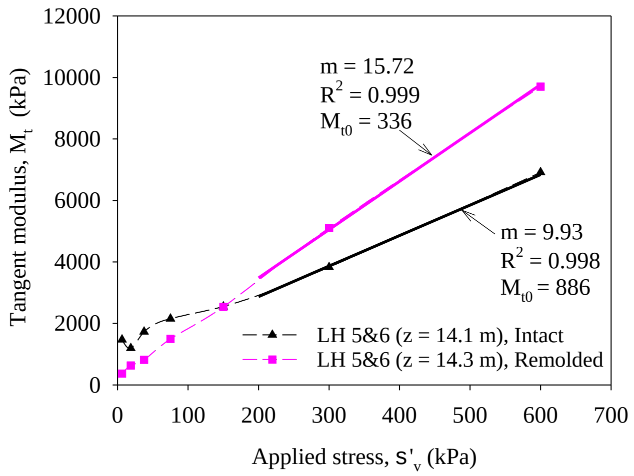



Figure 8. $M_t$ curves of intact and remolded specimens

Correlations between $m_{ms}$ and $m_{eq}$ and between $m_{ss}$ and $m_{eq}$ for the twelve remolded specimens are shown in Figures 9b and 9b, respectively. Interestingly, the ratio of $m_{eq}/m_{ms}$ becomes just 1.09 with better $R^2$ value. The ratio of $m_{eq}/m_{ss}$ is 1.0 with $R^2$ is 0.99, which is rather consistent with results of intact specimens shown in Fig. 5. This comparison indicate that the natural structure and heterogeneity of intact soil specimens have significant influence on *m* value determined from $M_t$ - $\sigma'_v$ curve. In fact, remolded soils are more homogeneous and less influenced by and natural structures. It might be concluded that, graphically, the $\varepsilon_v$ - $\sigma'_v$ curve is better for determining the *m* value from the consolidation test results.

Figure 10 shows a correlation between *m* obtained from $\varepsilon_v$ - $\sigma'_v$ curve and the porosity (*n*)

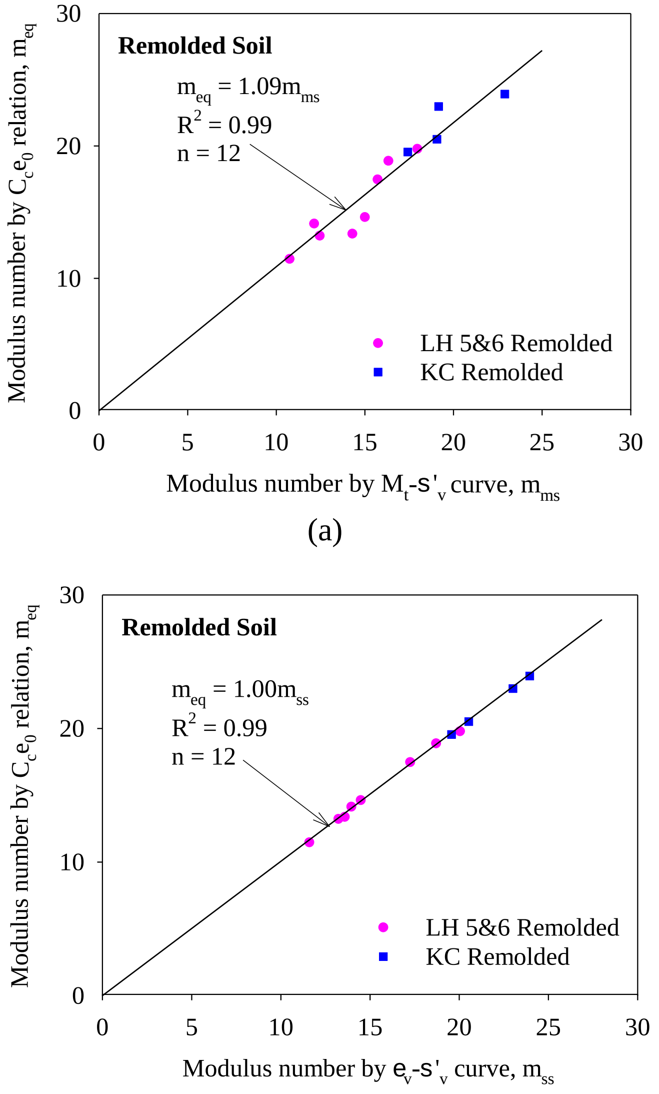



Figure 9. Correlation between: (a) $m_{ms}$ and $m_{eq}$; (b) $m_{ss}$ and $m_{eq}$

of the clay specimens in the RRD (*n* = 40 to 65%) with the inclusion of *m* from other soils in the world (Janbu, 1998). As shown, the data points of the RRD are slightly off the sketched range of other soil types. Generally, the *m* values of the RRD clays are similar to the range of Norwegian clays (*n* = 45 to 65%) and are distinctively larger than the values of Mexico clay (*n* = 70 to 95%).

### 4.2 Reference tangent modulus $E_{oed}^{ref}$

The reference tangent modulus ($E_{oed}^{ref}$) values of the data set were obtained by the graphical procedure (Figure 2). The $E_{oed}^{ref}$ values were found to range from 1.0 MPa through 6.0 MPa. It is interesting that this $E_{oed}^{ref}$ range of RRD clays is similar to the value range of Bangkok clay (Surarak et al. 2012), which varies from 1.0 to 5.5 MPa for soft to stiff clay.

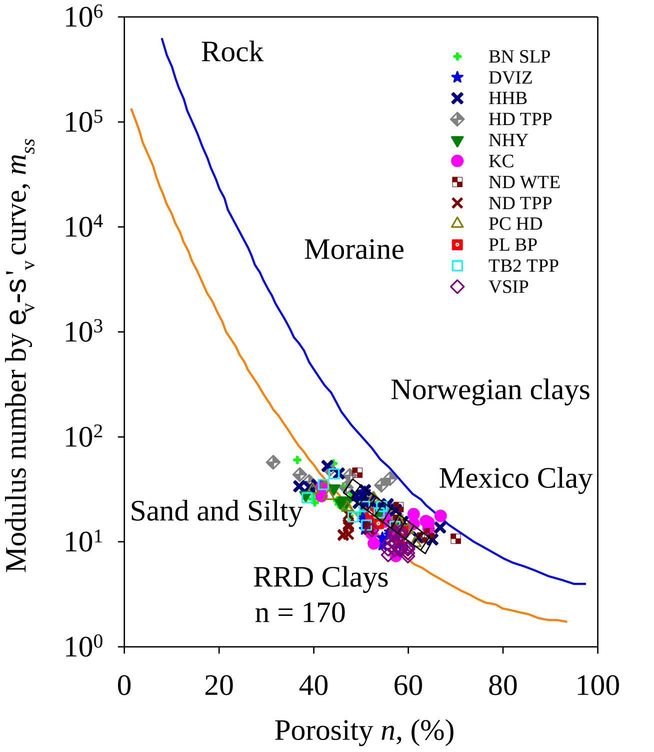



Figure 10. Correlation between $m$ and porosity ($n$) (modified after Janbu, 1998).

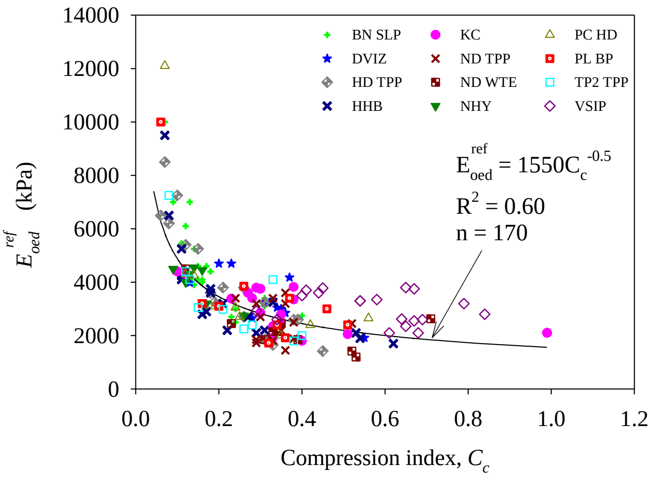



Figure 11. The correlation between $E_{oed}^{ref}$ and $C_c$

Attempts were made to correlate $E_{oed}^{ref}$ and other basic soil parameters obtained from the field and consolidation tests, e.g., the $q_t$, modulus number ($m$), compression index ($C_c$), etc. However, a clear correlation was only obtained for the $C_c$, as shown in Figure 11 and expressed by Eq. (5).

$$E_{oed}^{ref} = 1550\left(C_c\right)^{-0.50} \quad (5)$$

## 5. CONCLUSIONS

A database of consolidation test results from 12 investigation sites over the RRD was brought into analysis to depict the modulus number ($m$) and the reference tangent modulus ($E_{oed}^{ref}$). The following are key conclusions drawn from the study.

The $m$ value of the RRD clays was found to range mostly from 10 through 30, which agreed well with published values of soft to stiff clays. The $m$ value determined from $M_t$ - $\sigma'_v$ curve is on average 1.25 times larger than the value determined from $\varepsilon_v$ - $\sigma'_v$ curve. This larger ratio is attributed to the sensitive variation of $M_t$ with respect to the variation of $\Delta\varepsilon_v$ and also to the natural structure and heterogeneity of intact soil specimens. Notably, $m$ value from the $\varepsilon_v$ - $\sigma'_v$ curve is almost equal to the value determined by the equation $m = \ln10(1+e_0)/C_c$. This good agreement suggest that the $\varepsilon_v$ - $\sigma'_v$ curve be applied to determine $m$ value in practice rather than the theoretical $M_t$ - $\sigma'_v$ curve.

The $E_{oed}^{ref}$ value of the clays was found to range from 1.0 through 6.0 MPa, which is similar to the range of Bangkok clays. The $E_{oed}^{ref}$ value was found to have no clear correlation with corrected cone resistance ($q_t$) and modulus number ($m$), but a relatively good nonlinear correlation with $C_c$.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